

An alternative approach to ML estimation of multinomial choice models of voting

Alexey V. Zakharov

March 26, 2010

Abstract

I propose a new maximum likelihood methodology to estimate parameters of a multinomial choice model. I assume that there exist a set of agents whose choice sets are not observable, except for the choices actually made. One can extract additional information from data by assuming that the observed choices are a Nash equilibrium of some game. I use this method to reevaluate several probabilistic voting models. The estimated effect of policy platforms of parties on the vote is greater if one assumes the policy platforms to be endogenous.

Since Downs (1957), spatial voting models assume that the utility to a voter of choosing a candidate or political party is a function of the distance between the political agent's policy platform, and the voter's ideal policy. More recently, probabilistic voting models assumed that the utility of a voter is stochastic (Hinich, 1977, Hinich, Ledyard, and Ordeshook, 1972, Enelow and Hinich, 1989, McKelvey and Patty, 2006). Such models can be conveniently estimated from mass survey data using multinomial logit or probit, as the voter is described is represented by the same mathematical object in the formal probabilistic voting model, and in the corresponding econometric model. On the input, one usually has data from a pre-election survey, where each respondent indicates his socio-economic characteristics, his policy preferences on a set of issues, and the political party (or candidate) he intends to vote for. It remains to specify the functional form for voter utility and provide a voter ideal point for each observation in the survey, as well as to quantify the policy platform for each political agent. After that, the parameters of

the voter utility functions are estimated. In particular, one find how important to vote choice is the degree to which the ideal policy of a voter and the political agent’s policy differ. Such approach was first used by Poole and Rosenthal (1984) for US Presidential elections. Since then, it was used many times — for US Presidential elections (Alvarez and Nagler, 1998, Adams, Dow, and Merrill, 2006), Britain (Alvarez, Nagler, and Bowler, 2000, Quinn, Martin, and Whitford, 1999), Netherlands, Israel (Schofield and Sened, 2005), Turkey (Schofield, Gallego, Ozdemir, and Zakharov, 2009), Russia (Schofield and Zakharov, 2010), and others.

The ability to estimate the utility functions of the voters opens the possibility to numerically calculate Nash equilibrium policy positions in a “real” game of political competition with stochastic voters, and compare the resulting equilibrium policy positions with the ones observed in practice. If the political agents maximize expected share of the total vote, then under a broad range of assumptions one should expect a “convergent” equilibrium (Banks and Duggan, 2005), where political agents choose identical policy positions¹. A special case of this result is the “mean voter theorem” (Schofield, 2006, Lin, Enelow, and Dorussen, 1998); if the voter disutility from policy is quadratic, the political agents converge on the mean ideal policy position of the voters. Needless to say that this result is at a variance with the observed patterns of political competition; other, nonconvergent local equilibria do exist but still the estimated policy positions are far closer to each other than in the real life (Quinn and Martin, 2002, Schofield, 2006). Modifying the model, such as making the spatial coefficient depend on voter characteristics such as age or income (Zakharov and Fantazzini, 2008), or introducing voter abstention (Adams, Dow, and Merrill, 2006) does improve the fit between calculated equilibrium positions and observed positions; however, the discrepancy is not completely eliminated.

There are two principal ways to address the difference between the theoretical and the observed policy positions. One is to assume additional structure, such as policy-motivated agents (Duggan and Fey, 2005), political activism (Schofield, 2004), lobbying (Grossman and Helpman, 2002), endogenous valence (Zakharov, 2009, Ashworth and Bueno de Mesquita, 2005), incumbency advantage (Groseclose, 2001). The other way

¹Zakharov (2009) explores a model where the political agents are allowed to have generalized preferences over voting outcomes, instead of being risk-neutral. The convergence result is found not to hold).

is to produce a more realistic estimate of the model’s parameters by incorporating the existing structure of the formal model of political competition into the econometric model.

The principal assumption of a multinomial choice model is that there are a number of individuals whose properties are known; I also observe the choice of each individual, and the set of alternatives available to him. I can then construct a utility function that is associated with each of the outcomes, and estimate its parameters — usually through the maximum likelihood method (although sometimes different methods, such as Bayesian estimation, are used). This methodology ignores additional information that is sometimes available in the problem context. In a “spatial” voting model it is assumed that the voter’s utility toward a party is a function of the distances between the voter’s preferences on each issue, and the party’s stated positions on those issues (Poole and Rosenthal, 1984; Schofield and Sened, 2006). All previous research treated party positions as exogenous and arbitrary. However, from the problem context I know that each party cares about the share of vote that it receives. I know that the observed policy positions of each party is rational, conditional on the party’s utility function, the set of alternative policy positions available to the party, and the party’s knowledge of the voters’ decision model. That information is ignored in the traditional multinomial choice models, since the set of alternatives available to each party is not observable.

This paper proposes a methodology to incorporate this additional information in the likelihood function. In the first section, I postulate the assumptions used in this approach, and define the modified likelihood function. In the second section, I use it to re-evaluate a voting model estimated in Schofield (2006).²

1 The multinomial choice model and game definition.

I consider a problem of estimating a model of individual choice. I have a dataset with $i = 1, \dots, N$ observations, each corresponding to an individual. For each observation I have a vector of personal characteristics $x_i \in \mathbf{R}^{M_1}$, and a choice variable $d_i \in \{1, \dots, J\}$. I assume that the utility of individual i choosing an alternative j is

$$u_{ij} = u(x_i, \alpha_j, \beta, j) + \epsilon_{ij} = \bar{u}_{ij} + \epsilon_{ij}, \quad (1)$$

²Structural approach in the analysis of roll call voting was explored in Clinton and Meirowitz (2003).

where $\alpha_j \in \mathbf{R}^{M_2}$ is a vector of choice-specific parameters, and $\beta \in \mathbf{R}^{M_3}$ is a vector of choice-independent parameters. I make some assumptions about the distribution of the random variables ϵ_{ij} — usually independence for different values of i . Let $d \in J^N$ denote the choices of all individuals, and $x \in \mathbf{R}^{M_1 N}$ the personal characteristics of all individuals. Our goal is to estimate the values of the parameters $\alpha = (\alpha_j) \in \mathbf{R}^{M_2 J}$, β given our observations (x, d) .

A common way to solve this problem is through the maximum-likelihood method. For example, assume that ϵ_{ij} are distributed independently with a Type 1 extreme value distribution:

$$P(\epsilon_{ij} \leq h) = e^{-e^{-h}}. \quad (2)$$

Then, the likelihood of observation i would be

$$P_i = \frac{e^{\bar{u}_{id_i}}}{\sum_{k=1}^J e^{\bar{u}_{ik}}}, \quad (3)$$

and of the whole sample —

$$L(x, d, \alpha, \beta) = \prod_{i=1}^N P_i. \quad (4)$$

Maximizing L will give us the maximum-likelihood estimates of α and β . These are the parameter variables that maximize the probability that the observed outcome will arise in nature, given our assumptions about the form of the utility function, and the distribution of the error terms.³

I now introduce a new concept that will require a slight modification of the model.

Assumption 1 There exist K *players*. Each player k can choose some action y_k from a finite strategy set S_k before the individuals make their choices.

Put $S = \times S_k$. Let $y \in S$ denote an action profile for the players. For any k and $y \in S$, let y_{-k} be the actions of all players other than k .

³The maximum likelihood method has its well-known advantages and disadvantages. It is consistent and asymptotically unbiased; however, small-sample bias can be substantial, especially for those α_j where the number of individuals who have chosen alternative j is small. Comparing different models and estimating confidence intervals can also be a problem. An alternative approach is to treat the parameters as random variables, and estimate their joint distributions through a repeated Bayesian updating process.

Assumption 2 There are N individuals (or non-players). The payoff to an individual i choosing an alternative j depends on the actions of the players:

$$u_{ij} = u(x_i, \alpha_j, \beta, y, j) + \epsilon_{ij} = \bar{u}_{ij} + \epsilon_{ij} \quad (5)$$

Assumption 3 Every realization of d defines a payoff $U_k(d, y)$ to every player k , for every y . The players know the true values of the parameters (α, β) and x , but cannot observe ϵ_{ij} s.

The choices of the individuals d and hence the payoffs $U_k(d, y)$ are stochastic. However, I can define the expected payoffs, based on what I know about the utilities of individuals (5). Assuming that ϵ_{ij} are independent, the expected payoff of player k is

$$\bar{U}_k(x, \alpha, \beta, y) = \sum_{\delta \in J^N} \left(\prod_{i=1}^N p_{i\delta_i}(x_i, \alpha, \beta, y) \right) U_k(\delta, y). \quad (6)$$

where δ runs through all possible choice profiles, and $p_{i\delta_i}$ is the probability that individual i chooses alternative δ_i .

Now I can formulate the final assumption necessary in our methodology.

Assumption 4 The observed actions y are a Nash equilibrium in a game with players $1, \dots, K$, strategy sets S_k , and utilities

$$\tilde{U}_k = \bar{U}_k + \epsilon_{y_k}, \quad (7)$$

where ϵ_{y_k} are independent random variables. The values ϵ_{y_k} are known to player k , but not to the observer.

Speaking in the terms of this model, the traditional maximum likelihood method is about finding the parameters (α, β) that best explain the actions of non-players, given their characteristics x and observed choices d . A similar method can be used to find (α, β) that best explain the actions of players, conditional on the observed characteristics x of non-players, and the non-observed (but assumed) set of actions S available to the players.

Suppose that $y, y' \in S$ are two strategy profiles for the players, and I observe y . From Assumption 4 it follows that for every k , I have

$$\tilde{U}_k(x, \alpha, \beta, y) \geq \tilde{U}_k(x, \alpha, \beta, (y'_k, y_{-k})). \quad (8)$$

I want to find the estimates of α , β that best reflect the fact that every player k chose action y_k , given the actions y_{-k} of all other players. In order to find those estimates given existing data, I need to specify what alternative actions did each player k consider, before choosing y_k .

Consider two player action profiles, y and some y' . Denote by

$$P_k(x, \alpha, \beta, y, S_k) = P(\tilde{U}_k(x, \alpha, \beta, y) \geq \tilde{U}_k(x, \alpha, \beta, (y'_k, y_{-k})) \text{ for all } y'_k \in S_k - \{y_k\}) \quad (9)$$

the probability that player k chooses action y_k over all other actions $y'_k \in S_k$, given that all other players choose y_{-k} .

Suppose that $S = \times S_k$ is a set of action profiles, with $y \in S$. Suppose that player k knows that all other players will choose y_{-k} . The likelihood of observing $y \in S$ is then

$$L_P(x, \alpha, \beta, y, S) = \prod_{k=1}^K P_k(x, \alpha, \beta, y, S_k). \quad (10)$$

Now, I can give the definition of a new estimator for (α, β) .

Definition 1 Let $0 < \gamma \leq 1$, and S be a set of alternatives for players. The *weighted Nash equilibrium maximum likelihood estimator* of (α, β) maximizes the weighted likelihood function

$$L = L_P(x, \alpha, \beta, y, S)^\gamma L(x, d, \alpha, \beta)^{1-\gamma}. \quad (11)$$

2 Example — a probabilistic voting model.

Suppose that an individual corresponds to an observation in a sample of voters. Each observation includes a vector of personal characteristics (such as age or income) $x_i \in \mathbf{R}^{M_1}$, and of value $v_i \in \mathbf{R}^{M_4}$ that reflects the individual's preferences with respect to the policies that will be carried out by the winning party in the election. Note that as $K = J$, I will use subscript j to index players. The choice variable d_i represents the index of the political party that the individual intends to vote for in the upcoming election. Let the utility functions of the individuals be given by

$$u_{ij} = a_j + \alpha_j^T x_i + \beta \|v_i - y_j\|^2 + \epsilon_{ij} = \bar{u}_{ij} + \epsilon_{ij}, \quad (12)$$

where a_j is a party-specific constant, $\alpha_j \in \mathbf{R}^{M_1}$ is a party-specific vector of parameters, β is a parameter, $\|\cdot\|$ is the Euclidean norm, $y_j \in \mathbf{R}^{M_4}$ is the policy program of party

j , and ϵ_{ij} is an independent random variable. Values x_i are usually the socio-economic characteristics of the voter (age, religion, etc). Assuming the distribution (2), I have the probability of individual i voting for party j given by

$$p_{ij} = \frac{e^{\bar{u}_{ij}}}{\sum_{h=1}^J e^{\bar{u}_{ih}}}. \quad (13)$$

If I assume that the payoff of a political party is equal to the expected number of votes that it will receive in the elections times a constant μ_j , I have

$$U_j(x, \alpha, \beta, y) = \mu_j \sum_{i=1}^N p_{ij}. \quad (14)$$

For each j , this value is a function of individual characteristics x , the parameters α, β , and the policy platforms y . I can define a game between the J parties, where the strategy of party j is $y_j \in \mathbf{R}^{M_4}$, and the payoff is (6).

If ϵ_k is distributed with distribution (2), then I must have

$$L_P(x, \alpha, \beta, y, S) = \prod_{k=1}^K \frac{e^{U_j(x, \alpha, \beta, y)}}{\sum_{y'_j \in S_j - \{y_j\}} e^{U_j(x, \alpha, \beta, (y'_j, y_{-j}))}}. \quad (15)$$

Let the weighted log likelihood function for this problem be

$$L = w_V L_V + w_P L_P, \quad (16)$$

where L_V is the likelihood of the observed voting profile, and w_V, w_P are weights.

Consider a dataset for 1996 Israel Knesset elections and a two-dimensional spatial model analyzed in Schofield (2007). To simplify exposition, I take $\alpha_j = 0$; this corresponds to the “valence-only” model, where the voter’s utility toward a party does not depend on his socio-economic characteristics. I consider the two largest parties (Likud and Avoda) to be players. For each party, the strategy set has five elements: the observed policy position, and four deviations (plus or minus 1 on each dimension). I take $\mu_1 = \mu_2 = 5/N$. Let the weights be $w_V = (1 - \gamma)$ and $w_P = 600\gamma$. Hence, $\gamma = 0$ corresponds to the traditional maximum-likelihood approach; for $\gamma = 1$, only the likelihood of players (political parties in this case) is considered.

Table 1 shows the result of model estimation for different values of γ .

In this model, a voter has seven choices, each corresponding to a political party; the first six numbers in each column correspond to the parameter a_j for each of the choice

	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 0.8$
Likud	0.7778	0.6135	2.3978
Labor	0.9901	0.6552	1.8475
Mafdal	-0.6270	-1.0018	-0.8998
Modelet	-1.2595	-0.8874	1.7995
Third Way	-2.2916	-2.4721	-0.3101
Shas	-2.0239	-2.5701	-3.0521
β	-1.2075	-1.9050	-3.6245
Log likelihood (voters)	-776.95	-823.0	-1,204.5
Log likelihood (parties)	-1,444.1	-1,338.0	-1,172.8

Table 1: Estimation of valence only model with one β parameter

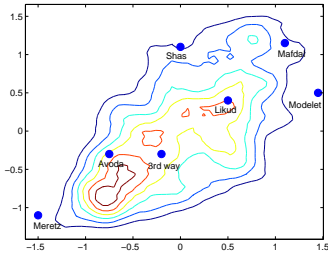
options (the seventh parameter is fixed at zero to identify the model). One can see that the more weight one puts on the likelihood of party policy positions, the higher is the estimated value of β , which reflects the importance of party policy positions in voter's decision.

The voter model estimated by the new methodology can produce greater consistency between observed and predicted behavior of political parties. Consider Figure 1. The first subfigure shows the actual positions of the seven political parties, superimposed on the density plot of voter policy preferences. The second and third figures show the simulated local Nash equilibria⁴, where the utility functions of the parties were assumed to be party vote shares.

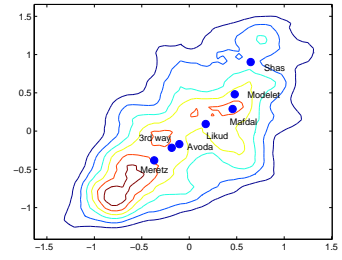
The equilibrium that used the voting model estimated for $\gamma = 0.5$ was more in line with the observed policy positions. In particular, the distance between the policy positions of the two largest parties (Likud and Avoda) is much larger in the third figure than in the second figure, although it is still smaller than the observed policy distance.

The outcome of estimation depends on our assumptions about the strategy sets available to the political parties. With a voter utility function (12) one should expect the estimates of β to be closer to the baseline estimate for $\gamma = 0$ if the variance of the elements of the policy sets S_k is smaller (as choosing a more proximate alternative policy

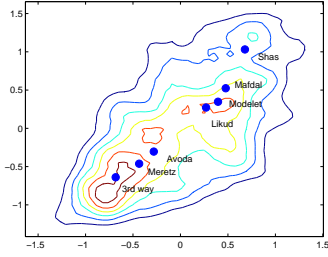
⁴A more general version of model (12) was used.



(a) Actual positions



(b) Nash equilibrium, $\gamma = 0$



(c) Nash equilibrium, $\gamma = 0.5$

Figure 1: Comparison of simulated Nash equilibria

will have less effect on the voteshare).

To check that hypothesis, I re-estimated the probabilistic voting model with different strategy sets. Again, I assumed that there are two parties that are players; the strategy set of each party consisted of its observed position, plus a number of normal, independent, zero-mean perturbations to that position. I repeated the estimation 100 times for each value of the standard deviation of the perturbation (0.5 or 1), and each size of strategy sets (6 or 11). Figure 2 shows the resulting densities of estimated parameters β .

There are two observations to make. First, the variance of elements in the strategy set has the predicted effect on the estimated value of β . Second, the variance of the estimates of β is smaller as the size of the strategy set increases.

3 Policy-motivated parties

The proposed framework can be used to estimate various parameters of the party utility functions. Suppose, for example, that the political parties are partly or fully motivated by policy, as in Wittman (1973), Calvert (1985), or Duggan and Fey (2005). The payoff to a political party is taken to be the weighted sum of the party's share of vote, and the

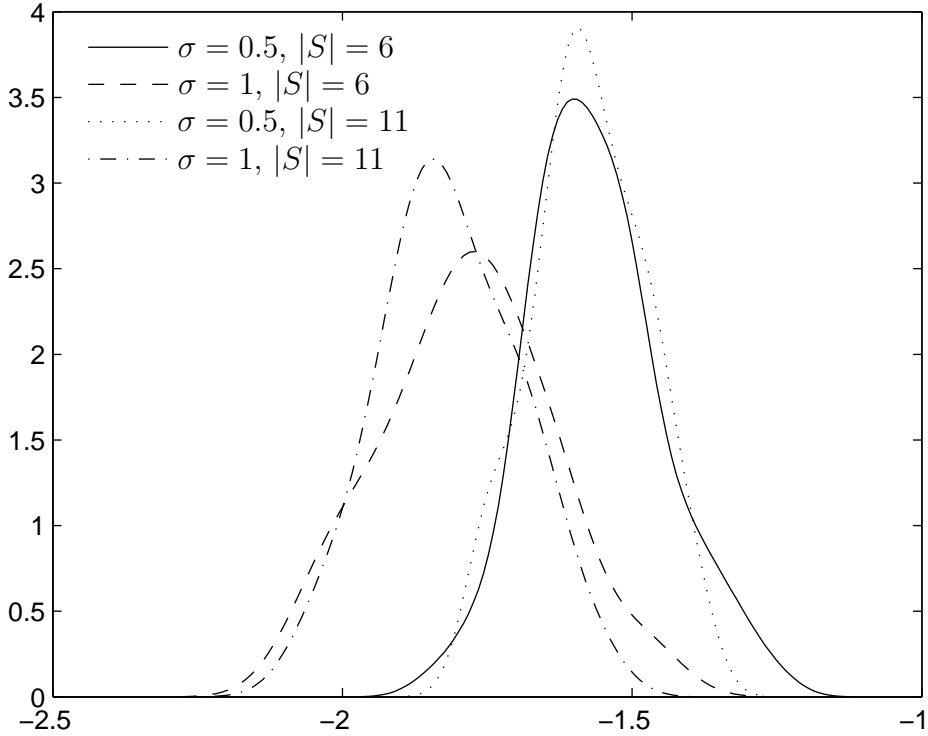


Figure 2: Distributions of β estimates, $\gamma = 0.5$.

its expected disutility from the policy that is implemented. I assume that the party's share of vote determines the probability with which the resulting policy outcome is equal to that party's policy position in an election. We have

$$U_j(x, \alpha, \beta, y, \lambda_j, a_j) = \mu_j \left(\lambda_j \sum_{i=1}^N p_{ij} - (1 - \lambda_j) \sum_{k=1}^K \sum_{i=1}^N p_{ik} \|y_k - a_j\|^2 \right), \quad (17)$$

where λ_j is the weight that party j places on holding office, and a_j is that party's best policy alternative.

I use the same dataset from 1996 Israel election. Take $w_V = (1 - \gamma)$, $w_P = 600\gamma$, and $\gamma = 0.5$. Let Likud and Avoda parties be players. The strategy set for each party is its observed position, plus 10 positions at distance of $d = 0.3$ from the observed position. The preferred position of each party is taken to be its observed position: $a_j = y_j$. Table 2 shows the results of the likelihood maximization.

This table tells us which λ best describes the observed policy positions of parties (and the observed survey response), given the assumption that the political parties know the true parameters of the voter utility function, and are maximizing (17). The first case

λ	β	$\ln(L)$		
	$a_1 = (0.5, 0, 4), a_2 = (-0.75, -0.3)$		$a_1 = (1, 0.8), a_2 = (-1.5, -0.6)$	
0	-0.9144	-972	-0.9939	-1639
0.1	-0.9224	-1014	-1.0462	-1656
0.2	-1.2542	-1050	-0.9756	-1674
0.3	-1.2191	-1096	-1.1057	-1691
0.4	-1.2625	-1142	-1.0569	-1708
0.5	-1.4225	-1191	-1.4467	-1730
0.6	-1.3142	-1239	-1.2161	-1741
0.7	-1.0606	-1296	-1.3278	-1758
0.8	-1.2030	-1343	-1.2673	-1772
0.9	-1.0751	-1400	-0.9593	-1794
1	-1.1339	-1453	-0.9958	-1809

Table 2: Estimated β for various λ for policy-motivated political parties, .

considered is $a_1 = (0.5, 0, 4)$, $a_2 = (-0.75, -0.3)$, which corresponds to the observed policy positions of the two parties. Under these assumptions the parties appear to be policy-motivated. The party's utility at the observed policy position (which is assumed to be its best policy) relative to the utility at the vote-maximizing position is highest if that party is policy-motivated. Taking $a_1 = (1, 0.8)$ and $a_2 = (-1.5, -0.6)$ does not change the result, with the same estimated β ; however, with $a_1 = (0, 0)$, $a_2 = (0, 0)$ the result is reversed: the most likely value is $\lambda = 1$. So, if we have a reason to believe that, all other things being equal, the parties prefer centrist policies, then, most likely, the parties are not policy motivated at all.

4 Conclusion

I propose a methodology to improve maximum likelihood estimates of multinomial choice models, by assuming the existence of additional utility-maximizing players. One cannot observe the choice sets of those players, only the choices that are actually made. I postulate that the observed choices maximize the utility of each player; by assuming some

strategy set for each player, one can construct a modified maximum likelihood estimator.

This approach is applied to estimation of a spatial voting model using the data from a survey conducted prior to 1996 Israel Knesset election. The common problem with all previously estimated spatial voting models is the inconsistency between the observed policy positions of political agents (parties or candidates) and numerically simulated Nash equilibria (given the assumption that the political agents maximize votes). It is always the case that the policy platforms in a simulated Nash equilibrium are located much closer to the center and to each other than the actual policy positions. My research shows that a large part of this discrepancy arises because the traditional multinomial choice approach (either using maximum likelihood or Bayesian methods) underestimates the effect of policy position on vote choice by assuming that the policy platforms are exogenous.

References

- [1] James Adams, Jay Dow, and Samuel Merrill III. 2006. "The political consequences of alienation-based and indifference-based voter abstention: Applications to Presidential Elections" *Political Behavior* 28(1):161-189
- [2] Alvarez, RM, and J. Nagler. 1998. "Economics, entitlements, and social issues: Voter choice in the 1996 presidential election". *American Journal of Political Science* 42(4): 1349–1363
- [3] Alvarez, Michael, Jonathan Nagler, and Shaun Bowler. 2000. "Issues, Economics, and the Dynamics of Multiparty Elections: The British 1987 General Election". *American Political Science Review* 94(1): 131–149
- [4] Ashworth, Scott, and Ethan Bueno de Mesquita. 2005. Valence competition and platform divergence. Princeton University Typescript, 2005
- [5] Banks, Jeffrey, and John Duggan. 2005. "Probabilistic Voting in the Spatial Model of Elections: The Theory of Office-Motivated Candidates." in David Austen-Smith

and John Duggan, eds., *Social Choice and Strategic Decisions*. Springer, New York, NY

- [6] Calvert, Randall. 1985. “Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence” *American Journal of Political Science* 29(1): 69–95
- [7] Joshua D. Clinton and Adam Meirowitz. 2003. “Integrating Voting Theory and Roll Call Analysis: A Framework” *Political Analysis* 11:4
- [8] Downs, Antony. 1957. *An Economic Theory of Democracy*. Hew York: Harper & Row
- [9] Duggan, John, and Mark Fey. 2005. “Electoral competition with policy-motivated candidates” *Games and Economic Behavior* 51(2): 490–522
- [10] Groseclose, Timothy. 2001. “A model of candidate location when one candidate has a valence advantage.” *American Journal of Political Science* 45(5): 862–886
- [11] Gene M. Grossman and Elhanan Helpman. 2002. *Special Interest Politics* Massachusetts Institute of Technology
- [12] Hinich, Melvin. 1977. “Equilibrium in spatial voting: The median voter result is an artifact.” *Journal of Economic Theory* 16: 208–219
- [13] Hinich, Melvin, Ledyard, John, and Ordeshook, Peter. 1972. “Nonvoting and the existence of equilibrium under majority rule.” *Journal of Economic Theory* 4: 144–153
- [14] Lin, Tse-Min, James Enelow, and Han Dorussen. 1999. “Equilibrium in Multicandidate Probabilistic Spatial Voting.” *Public Choice* 98: 59–82
- [15] McKelvey, Richard, and John W. Patty. 2006. “A Theory of Voting in Large Elections.” *Games and Economic Behavior*, 57(1): 155–180
- [16] Poole, Keith T., and Howard Rosenthal. 1984. “U.S. Presidential Elections 1968-1980: A Spatial Analysis.” *American Journal of Political Science* 28(2): 282–312

- [17] Quinn, Kevin M., Andrew D. Martin, and Andrew B. Whitford. 1998. "Voter Choice in Multi-Party Democracies: A Test of Competing Theories and Models." *American Journal of Political Science* 43(4): 1231–1247
- [18] Quinn, Kevin Q. and Andrew D. Martin. 2002. "An Integrated Computational Model of Multiparty Electoral Competition." *Statistical Science* 17(4): 409–419
- [19] Schofield, Norman. 2006. "The Mean Voter Theorem: Necessary and Sufficient Conditions for Convergent Equilibrium." *Review Of Economic Studies* 42: 27–50
- [20] Norman Schofield and Alexey Zakharov. 2010. "A stochastic model of the 2007 Russian Duma election" *Public Choice*
- [21] Norman Schofield, Maria Gallego, Ugur Ozdemir, and Alexey Zakharov. 2009. "Political Equilibria in a Stochastic Valence Model of Elections in Turkey" Typescript
- [22] Wittman, Donald. 1983. "Candidate motivation: A synthesis of alternative theories" *The American Political Science Review* 77(1): 142–157
- [23] Zakharov, Alexei V. 2009. "Candidate Location and Endogenous Valence." *Public Choice* 138(3-4)
- [24] Zakharov, Alexey, and Dean Fantazzini. 2008. "Idiosyncratic issue salience in probabilistic voting models: The cases of Netherlands, UK, and Israel". Typescript