

# Euclidean or city-block? Estimation of voter preferences in a multi-dimensional probabilistic voting model.

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## Abstract

The relationship between a voter's policy attitudes on various issues, the policy positions of political parties (or candidates) on those issues, and voting behavior has attracted much attention in both empirical and theoretical literature. A significant question is how do the voters substitute between the differences in policy positions on various issues. One popular hypothesis is that the voter treats policy differences on any two issues as perfect substitutes; this corresponds to the 'city-block' disutility metric in the policy space. The other hypothesis is that the voter uses the Euclidean metric to gauge her disutility. This article estimates probabilistic voting models for multi-party elections in a number of countries. The distance metric is assumed to be the Minkowskii metric (of which both city-block and Euclidean metrics are special cases, depending on the parameter), and the value of this parameter is estimated. It is shown that the actual voting behavior is closer to the Euclidean than to city-block hypothesis; in fact, the latter is statistically rejected.

## 1 Introduction

The spatial voting models, known since Downs (1957), assume that the voters have satiated preferences over the space of policy alternatives. The shape of voter preferences over the set of policy choices has significant implications for the policy positions and other aspects of candidate and political parties strategies. This is especially relevant if the policy space is multidimensional.

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If the number of policy dimensions is greater than one, there are two important aspects to a voter's utility function over the space of policy alternatives. First, it is the shape of the voter's indifference curves. The policy position of a candidate can be viewed by a voter as a package of anti-goods, with each anti-good corresponding to the distance between the voter's policy position and the candidate's policy position on some issue. The shape of a voter's indifference curves determines the marginal rate of substitution between those anti-goods. In the recent literature, two special cases received the greatest attention. The first one is the city-block metric, when the marginal rate of substitution between the two anti-goods is constant and, hence, the two anti-goods are perfect substitutes. The second case is Euclidean metric, where the MRS is equal to the ratio of the anti-goods, times some constant that reflects the relative salience of the two policy dimensions.

The second question is how fast does the voter's disutility increase with the policy distance. This aspect of the voter utility function becomes important in social choice setting and in game-theoretic models when voters have valences or non-policy utilities, in the spirit of Groseclose (2001) or Ansolabehere and Snyder (2000).

What is policy distance? The Hotelling (1929) model of horizontal differentiation interprets distance as physical distance; so does the numerous literature in economic geography. In the political setting, a few models actually derive the preferences over policy positions from the model primitives. The best-known example of an explicit derivation of single-peaked preferences over a one-dimensional set of alternatives is Metzler and Richards (1981). In their example, the economic state variable is the tax rate; the voters are differentiated by their income, and the tax proceeds are used by the government to procure public goods.

Several ways to measure policy positions and distance were employed in empirical models (Mair, 2001). If one has a mass survey, the policy positions of the respondents on policy issues (including the general left-right scale) can be measured as answers (usually, on the Likert scale) to specific questions concerning the respondent's attitude to policy. The answer to each question can be used as an estimate of policy position for a separate issue, or the answers can be aggregated through factor analysis. There are two ways to compute policy positions of political parties (and, hence, the individual policy distances). First, the survey may ask each respondent to evaluate a position of each party or candidate on each policy-related questions; in that case, the policy distances on each issue, for each voter and each candidate may be computed as the differences between the voter's reported position, and the position of the candidate (as reported by that voter, or its average across all voters). Second, the position of each candidate (or party)

may be evaluated through expert surveys. If a mass survey is used, the ideal policy position of a voter is usually her position on the Likert scale, or a linear combination of such.

The second way to measure party positions is through a systematized analysis of its official policy statements and manifestos. Such analysis can produce estimates of policy positions that can be compared across countries and time periods; however, mapping these positions in the same space as voter preferences remains a challenge. Finally, the policy positions of individual parliament members can be estimated through roll-call analysis (Poole and Rosenthal, 1997, Clinton and Meirowitz, 2003).

It is well-known since Plott (1968), McKelvey (1976), and Schofield (1983) that a majority-rule equilibrium does not generally exist for Euclidean and, more generally, smooth convex preferences if the number of dimensions is two or larger. Humpreys and Laver (2010)<sup>1</sup> have shown that the city-block preferences are qualitatively different. The majority-rule equilibria do generically exist, that is, the existence conditions on the voter ideal points for such equilibria are that of inequality. Moreover, their proof is constructive: the equilibrium is located at the dimension-by-dimension median. The existence conditions are always satisfied in the two-dimensional case, but the measure of voter preference profiles for which the majority-rule equilibrium exists quickly converges to zero as the number of dimensions increases. This result can be criticized on the grounds that the city-block preferences are non-generic, as the indifference curves are discontinuous at a finite number of points. This objection can be countered, however, by saying that the result really implies that if the preferences are close to city-block *and* if there is a small positive benefit of voting for the status-quo, then the core is non-empty for some nonzero-measure set of voter preference profiles.

In a recent work, Eguila (2010) produced an axiomatization of various spatial preference relations (Euclidean or city-block). He started by assuming that an individual's state of the world is described by a finite number of attributes, with a linear order over the set of values that each attribute can take. The question was whether, for a given profile of preference relations over the lotteries over the possible states of the world, there exists a mapping from the set of lotteries to a Euclidean space, such that the individual preferences over the latter space are Euclidean or city-block. It was shown that the necessary and sufficient conditions was the separability and single-peakedness of the individual preferences over each attribute.

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<sup>1</sup>Rae and Taylor (1971) and Wendell and Thorson (1974) first produced this result for a two-dimensional case; Katz and Nitzan (1977) have shown that majority-rule equilibria do not always exist if the number of dimensions is larger.

Methodologically, this work is similar to earlier logit or probit probabilistic voting models that used the Euclidean distance with squared penalty function. The source of data is a mass survey taken before a parliamentary election. Two-dimensional voter ideal policies are obtained from factor analysis of answers to number of questions about the policy preferences of the respondent. The policy positions of the political parties in the same space are obtained through expert opinion. It is assumed that the utility that the voter assigns to a political party depends on both the policy distance and on the voter's characteristics such as education or income. Such works were done for 1989 Netherlands election (Quinn and Martin, 2002), 1992 and 1996 Israeli elections (Schofield, Sened, and Nixon, 1998, Schofield and Sened, 2005, and Schofield, 2007), UK in 1987 (Quinn, MArtin, and Whitford, 1999), Argentine (Schofield and Cataife, 2007), 1999 and 2002 elections in Turkey (Schofield and Ozdemir, 2008), and 2003 election in Russia (Schofield and Zakharov, 2010). All works find significant effects for both socio-economic variables and policy distance. <sup>2</sup>

Like in our previous work (Zakharov and Fantazzini, 2010), we allow the salience of policy dimensions to be different, as well as to depend on individual voter characteristics. If the policy distance for each issue is included in the utility function separately, then the saliences of the issues (or the weights of the corresponding policy distances in each utility functions) usually are different, and sometimes statistically insignificant (Thurner and Eymann, 2000, Clarke et. al., 2005). Ansolabehere, Rodden and Snyder (2008) found that while the effect of the policy distance on each individual issue may be insignificant, aggregation of policy issues (such as through factor analysis) produces models where the effect of policy distance on voting is significant. Hellwig (2008a) in his study of 16 European democracies has shown that the traditional left-right policy dimension is perceived as less important by those who are employed in either services sector or the sector producing tradeable goods. The author argued that the policymakers have less impact on the globalized industries than they do on the producers of nontradeable commodities. In this and in Hellwig (2008b), the author argues that the inability of elites to affect economic policy reduces the salience of the left-right policy issues in favor of nonpolicy factors, such as party valence of religious or ethnic attachment.

There have been very few empirical works that measured the relative values that the voters put on different policy considerations. Grynaviski and Corrigan (2006) look at the same basic questions addressed in this work: which distance metric — Euclidean or city-block — best

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<sup>2</sup>Similar results were obtained for the two-candidate stochastic models of US elections, starting from Poole and Rosenthal (1984).

explains the observed patterns of voting. They used the ANES data gathered before the 1996 US Presidential election. Each of the two candidates was evaluated on five policy issues, each corresponding to a survey question; they estimated the salience of each policy issue. The authors argued that models that used the city-block metrics performed better than the Euclidean model.

There are several differences between our approach and that of Grynaviski and Corrigan. First, in our case we estimate a single multinomial-choice model, versus a separate OLS model estimated for each alternative by Grynaviski and Corrigan. The second difference is that in our model specification, unlike in Grynaviski and Corrigan, both Euclidean and city-block metrics are special cases that correspond to different values of the Minkowskii distance parameter. Therefore, in our case it is sufficient simply to compare the log-likelihoods for different values of the parameter, instead of comparing the performances of two different models through Bayesian or Akaike information criteria. Third, we estimated not only the parameter of the distance metric (which is responsible for the preferences being city-block or Euclidean), but also the penalty parameter, that is, for example, whether the voter disutility is linear or quadratic in policy distance. Grynaviski and Corrigan assumed squared disutility. Finally, in some model specifications we allow the saliences of different policy dimensions to be idiosyncratic and depending on various voter characteristics such as income and education.

The results obtained in this work are contrary to those obtained by Westholm (1997) in his study of 1989 Norwegian elections. In that work, the city-block metric with linear policy distance slightly outperformed the two competing models — the Euclidean metric with linear and squared policy distances. There are several differences between this work and Westholm's. First, he used a much larger number of dimensions (six policy issues plus the left-right self-placement), while assuming the saliences of all dimensions to be equal. Second, he used an OLS model, with the dependent variable being the thermometer rating of a particular party by a voter. Thus for each voter, the number of observations was equal to the number of political parties. We believe that using the nonlinear, multinomial choice model such as the logit is more appropriate. One of the reasons is that it can be integrated with a game-theoretic probabilistic voting model. Thus, the output of a multinomial logit estimation can be used to construct the utility functions (such as the expected number of votes or the expected probability of winning) for political parties, such as in Quinn and Martin (2002) or Schofield and Zakharov (2010). Finally, the OLS model is not entirely appropriate, as the dependent variable is limited to a certain range, which violates the homoscedasticity assumption.

Another earlier work that favored city-block preferences over Euclidean preferences is Enelow,

Mendell and Ramesh (1988), using the data from a pre-election wave of 1984 National Election Study. In many ways it is similar to Westholm (1997): the party thermometer ratings is the dependent variable, and a large number of policy dimensions is used, each corresponding to a specific question on the survey. Unlike Westholm, they did not assume that the saliences of policy dimensions are equal.

For all datasets that we study, the hypothesis that the preferences are city-block is rejected; this finding is robust with respect to a number of model specifications. At the same time, we generally cannot reject the hypothesis that the preferences are Euclidean. When the Minkowski parameter is actually estimated, we usually arrive at a utility function that corresponds to the policy distances being near-complements (instead of perfect substitutes, as with the city-block preferences). The hypothesis that the voter disutility is linear in policy distance is also rejected; the disutility function is estimated to be near-quadratic in policy distance.

## 2 The model.

### 2.1 The spatial probabilistic voting model.

Suppose that there are  $N$  voters and  $K$  political parties who compete for their votes. The parties play a one-shot game. The strategy of each party  $j$  is a promise to carry out some policy  $y_j \in \mathbb{R}^L$ , where  $L$  is the number of policy dimensions. Each voter votes for the party that gives her the highest utility. The utility of voter  $i$  voting for party  $j$  is given by

$$u_{ij} = a_j + \alpha_j \cdot x_i - \psi(v_i, y_j) + \epsilon_{ij} = \bar{u}_{ij} + \epsilon_{ij}. \quad (1)$$

Here,  $a_j$  is the valence of party  $j$ ,  $x_i$  is the vector of voter  $i$ 's socio-economic characteristics (such as income, age, or ethnicity),  $\alpha_j$  is the vector of party-specific parameters, and  $\epsilon_{ij}$  is the unobserved random variable. It is assumed that  $\epsilon_{ij}$  has a continuous distribution that is everywhere defined. The function  $\psi(v, y)$  is the disutility that voter with a policy position  $v$  suffers if policy position  $y$  is indeed implemented.

Denote by

$$P_{ij} = P(u_{ij} = \max_k u_{ik}) \quad (2)$$

the probability that voter  $i$  votes for party  $j$ .

Given mass survey data, probabilistic voting models can be estimated using the maximum likelihood method. For example, assuming that the error terms  $\epsilon_{ij}$  are independent and iden-

tically distributed according to the Type 1 extreme value distribution

$$P(\epsilon \leq x) = e^{-e^{-x}}, \quad (3)$$

the likelihood of observation  $i$  can be conveniently written out in closed form

$$L_i = \frac{e^{\bar{u}_i d_i}}{\sum_j e^{\bar{u}_i j}}, \quad (4)$$

where  $d_i$  is the index of the party for which voter  $i$  intended to vote, and the likelihood  $L$  of the entire sample is the product of individual likelihoods. Maximum likelihood estimation involves maximizing  $L$  with respect to parameters  $\alpha_j$ ,  $a_j$ , and, possibly, the parameters of function  $\psi(\cdot, \cdot)$ .

## 2.2 The Minkowskii distance.

We now define the disutility function  $\psi(\cdot, \cdot)$ . The generalized Minkowski distance between policy alternatives  $v$  and  $y$  is defined as

$$d(v, y) = \left( \sum_{k=1}^K |\beta_k (y_k - v_k)|^\rho \right)^{\frac{1}{\rho}}, \quad (5)$$

where  $\rho > 0$ ,  $y_k$ ,  $v_k$  denote the  $k$ th components of  $y$  and  $v$ , respectively, and  $\beta_k$  is the salience of policy dimension  $k$ . The disutility function  $\psi(\cdot, \cdot)$  is taken to be

$$\psi(v, y) = d(v, y)^\gamma, \quad (6)$$

where  $\gamma > 0$  is the penalty parameter that reflects how quickly disutility increases with distance.

The Minkowskii distance has several attractive properties. First, it is scale-invariant: for all  $\alpha > 0$ , we have  $d(\alpha v, \alpha y) = \alpha d(v, y)$ . Second, both the Euclidean metric and the Manhattan metric are special cases of this function (corresponding to  $\rho = 2$  and  $\rho = 1$ , respectively). For  $\rho > 1$ , this metric generates strictly convex contour sets; the limiting case with  $\rho = \infty$  is known as the Chebyshev distance.

Each policy platform  $y$  can be considered by a voter with the ideal position  $v$  as a bundle of  $L$  anti-goods  $g_k = |v_k - y_k|$ . Each anti-good is the voter's disagreement with the party's position on some policy issue. The Minkowskii distance can be rewritten in terms of  $g = (g_1, \dots, g_L)$ :

$$\bar{d}(g) = \left( \sum_{k=1}^K \beta_k g_k^\rho \right)^{\frac{1}{\rho}}. \quad (7)$$

This function resembles the constant elasticity of substitution utility function.

If  $k_1$  and  $k_2$  are policy dimensions, the marginal rate of substitution between  $g_{k_1}$  and  $g_{k_2}$  is given by

$$MRS_{k_1, k_2} = -\frac{\frac{\partial \bar{d}}{\partial g_{k_1}}}{\frac{\partial \bar{d}}{\partial g_{k_2}}} = -\frac{\beta_{k_1} g_{k_1}^{\rho-1}}{\beta_{k_2} g_{k_2}^{\rho-1}}. \quad (8)$$

For  $\rho = 1$ , the marginal rate of substitution between the anti-goods is constant, so they are perfect substitutes. If  $\rho > 1$ , the marginal rate of substitution will depend on the ratio  $g_{k_1}/g_{k_2}$ . Suppose that  $g_{k_1} > g_{k_2}$ . As the value of the parameter  $\rho$  increases, the voter is willing to accept larger and larger quantities of antigood  $k_2$  (the policy distance on dimension  $k_2$ ), in order to reduce the quantity of antigood  $k_1$  by some amount. For Euclidean preferences, the MRS will be equal to the inverse of the ratio  $g_{k_1}/g_{k_2}$ . Finally, for  $\rho = \infty$ , the marginal rate of substitution is zero if  $g_{k_1} < g_{k_2}$  and is infinite if  $g_{k_1} > g_{k_2}$ . In that case the voter's disutility is determined by the largest distance on any policy dimension. These preferences are similar to the Leontief preferences, where the utility is equal to the smallest quantity of the positive goods consumed. For  $\rho = \infty$ , the disagreements on different policy dimensions will be perfect complements.

The indifference curves for a voter with the ideal point located at the origin are shown on Figure 1.

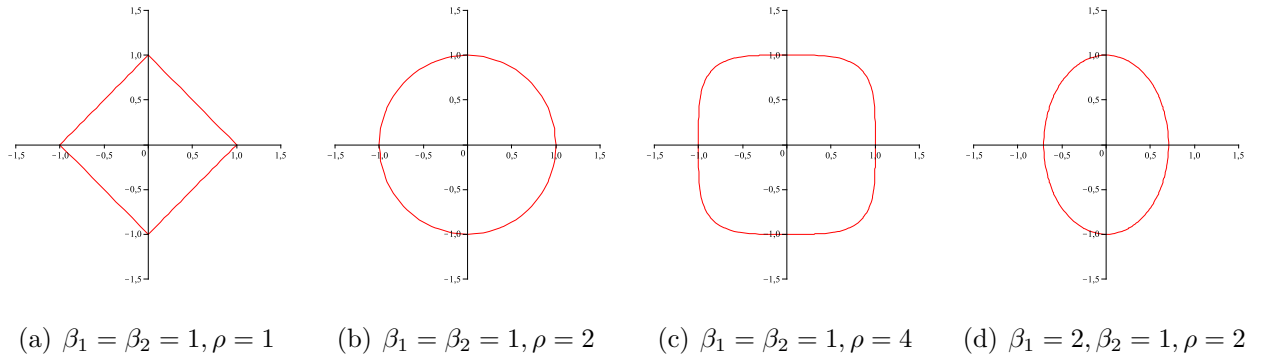


Figure 1: Voter indifference curves for the Minkowskii distance

In Figure 1(a), the policy distances are substitutes; in Figure 1(c) they are near complements, while 1(b) represents the Euclidean distance — the intermediate case for which the marginal rate of substitution between the anti-goods is equal to their ratio. In the final subfigure, the salience of the first policy dimension is greater than that of the second dimension.

The main hypotheses with respect to the substitution between the distances on various policy issues can be formulated as follows:

**Hypothesis 1.** The policy distances on different policy dimensions are perfect substitutes:

$$\rho = 1.$$

**Hypothesis 2.** The marginal rate of substitution between the policy distances on different policy dimensions is equal to their ratio:  $\rho = 2$ .

In addition, we test whether the voters have constant marginal utility with respect to policy distance.

**Hypothesis 3.** The voter’s disutility is linear in the distance to the party’s policy platform:  
 $\gamma = 1$ .

### 3 Empirical analysis.

#### 3.1 The data.

We use mass survey datasets from 4 countries: United Kingdom, Poland, Russia, and Netherlands. Each dataset was obtained in a year preceding parliamentary elections. For each country we select a list of political parties with significant electoral support. We discard every observation where the respondent intended to vote for a small party, or abstain, due to the well-known difficulty of estimating multinomial choice models with small number of observations for the dependent variable. Table 1 shows the list of datasets that we use, including the election year, the list of political parties and the number of observations used in the estimation, and the list of socio-economic variables.

Country	Year	$N$	Parties	Socios
Poland	2005	918	Law and Justice, Civic Platform, Self Defence, Left Alliance, Polish Families, Social Democracy, Democratic Party	age, gender, religion, education, communist
Netherlands	1977	529	PvdA, CDA, D66, VVD	religious, income, manual laborer, township size, education
United Kingdom	1979	426	Liberal, Conservative, Liberal Democrats	religious, income, manual laborer, township size, education
Russia	2007	1004	United Russia, LDPR, CPRF, Fair Russia	age gender income education, efficacy, Putin approval

Table 1: Summary of the survey data.

For each dataset, the ideal policies of the respondents were extracted by the application of factor analysis or principal component analysis to a list of questions discerning the respondent’s attitudes to a number of policy-related issues. For the Poland dataset, the ideal point estimates were taken from Tavits, Schofield and Ozdemir (2011). For Netherlands and the UK dataset

we used the same ideal policy estimates as in Quinn, Martin, and Whitford (1999). For Russia, the estimates were first obtained by Schofield and Zakharov (2010). The policy positions of the political parties were taken from the same sources as above. To obtain party policy positions for Poland, the authors applied expert opinion to the same lists of policy-related questions as in the corresponding mass surveys, then transformed the answers into the space of voter ideal policies by using the factor loadings from the output of the principal component analysis. The Netherlands and UK party positions were obtained through a survey of party elites. For Russia, the policy position for a party was taken to be the mean of the ideal policies of those respondents who intended to vote for that party<sup>3</sup>.

## 4 Estimation results.

For each country, we estimated the probabilistic voting model under several sets of restrictions. First, the model was estimated under unrestricted  $\rho$  and  $\gamma$ . Then, the model was estimated for various restrictions on  $\rho$  and  $\gamma$  (there were 6 sets of restrictions). The procedure was repeated for each country, for three sets of models. First, we assumed that the saliences of both policy dimensions were equal and constant across the voters. This corresponds to the spatial voting models estimated in most of the previous literature, including Tavits, Schofield and Ozdemir (2011), Quinn, Martin, and Whitford (1999), and Schofield and Zakharov (2010). Second, we assumed the saliences to be different, but constant across the voters. Finally, we assumed the saliences to be equal, but depend on voter socio-economic characteristics. In total, we estimated  $4 \times 3 \times 6 = 72$  different models. The part of the results relevant to testing Hypotheses 1-3 are shown on Table 2.

The first two columns show the ML estimates of  $\rho$  and  $\gamma$  when both parameters are unrestricted. For all four countries, in all three model families, the ML estimates of  $\rho$  were above 2, which suggests that the policy distances are complements rather than substitutes. For the case of UK, the estimated shape of the indifference curves is very close to the Chebyshev metric which corresponds to policy distances being perfect complements. The last six columns show the log likelihoods for models under various sets of restrictions on  $\gamma$  and  $\rho$ . For one model (Poland, variable salience,  $\gamma = 1$ ,  $\rho = 1$ ) we could not obtain the ML estimates. All values of  $\gamma$  lie between 1.44 and 1.97, which suggests increasing marginal disutility due to policy distance.

Under both assumptions about  $\gamma$ , imposing the restriction  $\rho = 1$  significantly reduces the

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<sup>3</sup>See Turina (2011) for a criticism of this approach.

		Estimated values		Log likelihoods					
		$\rho^*$	$\gamma^*$	$\rho = 1,$ $\gamma = 1$	$\rho = 2,$ $\gamma = 1$	$\rho = 1,$ $\gamma$ unrest.	$\rho = 2,$ $\gamma$ unrest.	$\rho$ unrest., $\gamma = 1$	$\rho$ unrest., $\gamma$ unrest.
$\beta_{i1} = \beta$ $\beta_{i2} = \beta$	Poland, 2005	2.35	1.44	-1407.01	-1402.88	-1406.25	-1401.04	-1402.76	-1400.90
	Netherlands, 1977	2.72	1.64	-444.33	-437.91	-441.22	-434.28	-437.05	-433.87
	UK, 1979	4.65	1.79	-366.46	-364.46	-365.61	-363.02	-363.97	-362.49
	Russia, 2007	2.99	1.75	-717.89	-709.92	-713.93	-704.74	-708.25	-703.77
$\beta_{i1} = \beta_1$ $\beta_{i2} = \beta_2$	Poland, 2005	2.49	1.57	-1404.53	-1399.80	-1402.98	-1396.89	-1399.54	-1396.60
	Netherlands, 1977	2.76	1.69	-442.86	-437.91	-439.71	-434.04	-437.00	-433.49
	UK, 1979	3.61	1.86	-363.62	-363.02	-361.60	-361.33	-362.89	-361.19
	Russia, 2007	2.99	1.75	-717.88	-709.90	-713.91	-704.73	-708.24	703.77
$\beta_{i1} = \beta_i$ $\beta_{i2} = \beta_i$	Poland, 2005	2.08	1.67	Not Conv.	-1395.60	1396.97	-1391.88	-1395.60	-1391.87
	Netherlands, 1977	2.84	1.97	-436.82	-429.75	-432.31	-423.67	-428.55	-422.97
	UK, 1979	4.65	1.79	-363.16	-361.19	-362.54	-359.82	-360.37	-358.73
	Russia, 2007	2.63	1.87	-715.36	-708.66	-710.93	-702.91	-707.61	-702.40

Table 2: Estimated values of  $\rho$  and  $\gamma$  and log likelihoods.

fit of the model. Table 3 shows the results of the likelihood ratio test on various restrictions on  $\rho$  and  $\gamma$ . We can see that the hypothesis that  $\rho = 1$  is rejected both when we assume  $\gamma = 1$  and an unrestricted  $\gamma$ .

		Hypothesis 1		Hypothesis 3		Hypothesis 4		
		$\rho = 1, \gamma = 1$	$\rho = 1, \gamma$ un.	$\rho = 2, \gamma = 1$	$\rho = 2, \gamma$ un.	$\rho = 1, \gamma = 1$	$\rho = 2, \gamma = 1$	$\rho$ un., $\gamma = 1$
	$H_0$ (Null):	$\rho = 1, \gamma = 1$	$\rho = 1, \gamma$ un.	$\rho = 2, \gamma = 1$	$\rho = 2, \gamma$ un.	$\rho = 1, \gamma = 1$	$\rho = 2, \gamma = 1$	$\rho$ un., $\gamma = 1$
	$H_A$ (Alt.):	$\rho$ un., $\gamma = 1$	$\rho$ un., $\gamma$ un.	$\rho$ un., $\gamma = 1$	$\rho$ un., $\gamma$ un.	$\rho = 1, \gamma$ un.	$\rho = 2, \gamma$ un.	$\rho$ un., $\gamma$ un.
$\beta_{i1} = \beta$ $\beta_{i1} = \beta$	Poland, 2005	8.50 (0.00)	10.70(0.00)	0.28 (0.59)	0.28 (0.60)	1,52 (0.22)	3.86 (0.06)	3.72(0.05)
	Netherlands, 1977	14.56 (0.00)	14.70 (0.00)	0.82 (0.36)	6.36 (0.01)	6.22 (0.01)	7.26 (0.01)	6.36 (0.01)
	UK, 1979	4.98 (0.03)	6.24 (0.01)	1.04 (0.30)	2.96 (0.09)	1.7 (0.19)	2.88 (0.09)	2.97 (0.09)
	Russia, 2007	19.28 (0.00)	20.32 (0.00)	1.94 (0.16)	8.96 (0.00)	7.94 (0.00)	10.36 (0.00)	8.96(0.00)
$\beta_{i1} = \beta_1$ $\beta_{i1} = \beta_2$	Poland, 2005	9.98 (0.00)	12.76(0.00)	0.58 (0.44)	0.58 (0.46)	3.1 (0.08)	5.82 (0.02)	5.88(0.02)
	Netherlands, 1977	11.72 (0.00)	12.44 (0.00)	1.12 (0.29)	7.02 (0.01)	6.3 (0.01)	7.44 (0.01)	7.02 (0.01)
	UK, 1979	1.46 (0.22)	0.82 (0.37)	0.28 (0.59)	3.40 (0.07)	4.04 (0.04)	3.58 (0.06)	3.40 (0.07)
	Russia, 2007	19.28 (0.00)	20.28(0.00)	1.92 (0.16)	1.92 (0.17)	7.94 (0.00)	10.34 (0.00)	8.94(0.00)
$\beta_{i1} = \beta_i$ $\beta_{i2} = \beta_i$	Poland, 2005	/	10.20(0.00)	0.02 (0.88)	0.02 (0.89)	/	7.46 (0.02)	7.44(0.01)
	Netherlands, 1977	16.54 (0.00)	18.68 (0.00)	1.4 (0.23)	1.40 (0.24)	9.02 (0.00)	12.16 (0.00)	11.16 (0.00)
	UK, 1979	5.58 (0.02)	7.62 (0.01)	2.1 (0.14)	2.18 (0.14)	1.4 (0.24)	2.74 (0.10)	3.28 (0.07)
	Russia, 2007	15.50 (0.00)	17.06(0.00)	1.02 (0.31)	1.02 (0.31)	8.88 (0.00)	11.5 (0.00)	10.42(0.00)

Table 3: Likelihood ratio tests

On the whole, we could not reject the hypothesis that  $\rho = 2$  and the voter preferences are Euclidean. We can see that Hypothesis 2 is rejected only if we assume unrestricted  $\gamma$ , and even then not for all datasets and model specifications. In particular, if we assume that the two policy issues have the same salience that depends on individual characteristics, the Euclidean preferences hypothesis holds. Finally, Hypothesis 3 is rejected. For  $\rho = 1$ ,  $\rho = 2$  and undrestricted  $\rho$ , removing the restriction that  $\gamma = 1$  significantly increases the fit of the model.

The multinomial logit model assumes that the idiosyncratic utility shocks  $\epsilon_{ij}$  are identical and independently distributed. We test this assumption using two methods. The first one is the test proposed by Hausman and McFadden (1984). The test statistic is computed from the estimated coefficients, and from the coefficients estimated from a restricted model, when one of the outcome categories is eliminated. Under the IIA assumption, the resulting test statistic has a chi-square distribution with the degrees of freedom equal to the number of parameters in the model. The test statistic can also be negative, which indicates that IIA holds. The values of the test statistics are shown in the central column on Table 4. We can see that in general, the IIA assumption holds.

	<b>Omitted Party</b>	Hausman test	Small-Hsiao test
Netherlands	Labor	62.64(0.00)**	6.14 (0.41)
	Liberal	9.50 (1.00)	7.94 (0.24)
	Christian Democrat	-68.61	4.74 (0.58)
	D66	59.19(0.00)**	125.91(0.00)**
Poland	League of Polish Families	6.81(1.00)	22.89(0.00)**
	Democratic Party	-18.58	2.30 (0.89)
	Social Democracy	23.65(1.00)	2.59 (0.86)
	Law and Justice	206.71(0.00)**	5.17 (0.52)
	Democratic Left Alliance	-1.88	3.77 (0.71)
	Civic Platform	48.93(0.52)	7.92 (0.24)
	People's Party	-28.05	2.02 (0.92)
	Self-Defence	-58.90	2.92 (0.82)
Russia	United Russia	-90.43	16.44(0.02)*
	Communists	12.63(1.00)	8.32 (0.31)
	Fair Russia	-0.37	8.32 (0.31)
	LDPR	46.96(0.03)*	25.46(0.00)**
UK	Conservative	-17.66	1.03(0.98)
	Liberal Democrat	-2.27	4.07 (0.67)
	Labor	-900.09	2.95 (0.82)

Table 4: Hausman and Small-Hsiao tests of the IIA Assumption. \*\* — IIA rejected at 1% level. \* — IIA rejected at 5% level.

We also test the assumption that the error terms  $\epsilon_{ij}$  are independently distributed across choices. This IIA assumption can be tested for by calculating the Small-Hsiao test statistics (Small and Hsiao, 1985). Under the IIA assumption, a Small-Hsiao statistic is distributed as chi-square with the degrees of freedom equal to the number of the parameters plus one. Generally, the IIA assumption holds very well for Poland and UK and less so for Russia and Netherlands.

Tables 5, 6, 7, and 8 show the detailed results of estimation for the models with unrestricted

$\rho$  and  $\gamma$ . The  $\gamma$  coefficient is significantly different from zero; the  $\rho$  coefficient is significantly different from zero for all countries except the UK.

In the second column, the saliences of both policy dimensions are equal, but differ across voters, depending on their socio-economic characteristics. For Poland, Netherlands, and the UK, the salience of the policy dimensions positively depend on the person's education, suggesting that more educated voters are more sensitive to party policy positions than their less educated compatriots; this result is in line with Zakharov and Fantazzini (2010).

We checked the robustness of the results by estimating the spatial-only models, where the voter's utility toward a party did not depend on the voter's socio-economic characteristics. The partial results are shown in the fourth columns of Tables 5, 6, 7, and 8. Although the fit of the model is in all cases reduced by a large amount (as expected), the estimated values of  $\rho$  and  $\gamma$  did not differ significantly from those for the previous three models.

## 5 Conclusion.

The purpose of this work was to determine what kind of cognitive distance metrics are most appropriate to model political decision making. We estimate a nonlinear multinomial choice model on a number of datasets. Contrary to previous studies, that used thermometer ratings as the dependent variable, we find that the preferences of the voters more closely correspond to Euclidean than to city-block distance metrics. Our results are robust with respect to a number of model specifications. The fact that the measured preferences of the voters are not city-block suggests that we should be more careful in expecting the existence of a majority-rule equilibrium that is invariant with respect to different political institutions.

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		$\beta_{ki} = \bar{\beta}_k$		$\beta_{ki} = \bar{\beta}_k$	
		$\beta_1 = \beta_2$	$\beta_1 = \beta_2$	$\beta_1 \neq \beta_2$	$\beta_1 \neq \beta_2$
Labor	<code>_const</code>	1.92 (0.00)	1.77 (0.00)	1.91 (0.00)	1.66 (0.00)
	<code>manlab</code>	1.50 (0.02)	1.37 (0.04)	1.50 (0.03)	
	<code>relig</code>	0.07 (0.65)	0.09 (0.60)	0.07 (0.67)	
	<code>income</code>	-0.05 (0.24)	-0.04 (0.31)	-0.05 (0.26)	
	<code>stown</code>	0.36 (0.16)	0.36 (0.18)	0.36 (0.16)	
	<code>educ</code>	-0.20 (0.00)	-0.18 (0.01)	-0.20 (0.00)	
Liberal	<code>_const</code>	0.28 (0.72)	-0.09 (0.93)	0.30 (0.70)	1.21 (0.00)
	<code>manlab</code>	-0.43 (0.64)	-0.41 (0.66)	-0.44 (0.63)	
	<code>relig</code>	0.04 (0.85)	0.05 (0.82)	0.02 (0.94)	
	<code>income</code>	0.10 (0.04)	0.11 (0.04)	0.10 (0.04)	
	<code>stown</code>	0.20 (0.53)	0.22 (0.53)	0.19 (0.54)	
	<code>educ</code>	-0.04 (0.53)	0.02 (0.75)	-0.04 (0.60)	
Christian Democrat	<code>_const</code>	-1.34 (0.10)	-1.67 (0.05)	-1.30 (0.11)	1.56 (0.00)
	<code>manlab</code>	1.19 (0.10)	1.13 (0.13)	1.15 (0.12)	
	<code>relig</code>	1.40 (0.00)	1.43 (0.00)	1.38 (0.00)	
	<code>income</code>	0.02 (0.67)	0.03 (0.59)	0.02 (0.63)	
	<code>stown</code>	0.53 (0.07)	0.54 (0.08)	0.52 (0.08)	
	<code>educ</code>	-0.14 (0.02)	-0.08 (0.26)	-0.14 (0.03)	
$\beta_1$	<code>_const</code>	0.95 (0.00)	0.63 (0.00)	0.94 (0.00)	0.97 (0.00)
	<code>manlab</code>		-0.18 (0.20)		
	<code>relig</code>		0.00 (0.95)		
	<code>income</code>		0.00 (0.78)		
	<code>stown</code>		-0.01 (0.91)		
	<code>educ</code>		0.07 (0.01)		
$\beta_2$	<code>_const</code>			1.08 (0.00)	1.35 (0.00)
$\rho$	<code>_const</code>	2.72 (0.02)	2.84 (0.01)	2.76 (0.01)	2.61 (0.00)
$\gamma$	<code>_const</code>	1.64 (0.00)	1.97 (0.00)	1.69 (0.00)	1.74 (0.00)
Log likelihood		-433.87	-422.97	-433.49	-535.83
N		529	529	529	529

Table 5: Estimated coefficients with unrestricted  $\rho$ ,  $\gamma$  for Netherlands.

		$\beta_{ki} = \bar{\beta}_k$		$\beta_{ki} = \bar{\beta}_k$	
		$\beta_1 = \beta_2$	$\beta_1 = \beta_2$	$\beta_1 \neq \beta_2$	$\beta_1 \neq \beta_2$
Conservative	<code>_const</code>	0.57 (0.23)	0.58 (0.25)	0.50 (0.30)	-0.11 (0.41)
	<code>manlab</code>	1.05 (0.00)	0.99 (0.00)	1.07 (0.00)	
	<code>relig</code>	-0.24 (0.03)	-0.23 (0.05)	-0.24 (0.02)	
	<code>income</code>	-0.10 (0.05)	-0.11 (0.05)	-0.10 (0.05)	
	<code>stown</code>	0.08 (0.62)	0.05 (0.74)	0.07 (0.67)	
	<code>educ</code>	0.04 (0.41)	0.08 (0.18)	0.05 (0.37)	
Liberal Democral	<code>_const</code>	-0.87 (0.18)	-0.93 (0.15)	-0.90 (0.16)	-1.61 (0.00)
	<code>manlab</code>	0.39 (0.33)	0.38 (0.34)	0.40 (0.31)	
	<code>relig</code>	0.00 (0.99)	0.02 (0.90)	0.00 (1.00)	
	<code>income</code>	-0.11 (0.11)	-0.11 (0.13)	-0.11 (0.11)	
	<code>stown</code>	-0.25 (0.26)	-0.25 (0.26)	-0.25 (0.26)	
	<code>educ</code>	0.12 (0.05)	0.12 (0.06)	0.12 (0.05)	
$\beta_1$	<code>_const</code>	0.59 (0.00)	0.63 (0.00)	0.58 (0.00)	0.58 (0.00)
	<code>manlab</code>		-0.05 (0.64)		
	<code>relig</code>		0.00 (0.95)		
	<code>income</code>		-0.01 (0.52)		
	<code>stown</code>		0.01 (0.94)		
	<code>educ</code>		0.04 (0.08)		
$\beta_2$	<code>_const</code>			0.39 (0.06)	0.38 (0.05)
$\rho$	<code>_const</code>	4.65 (0.55)	178.85 (0.90)	3.62 (0.48)	2.51 (0.24)
$\gamma$	<code>_const</code>	1.79 (0.00)	1.89 (0.00)	1.86 (0.00)	1.80 (0.00)
Log likelihood		-362.49	-358.73	-361.19	-376.4
N		426	426	426	426

Table 6: Estimated coefficients with unrestricted  $\rho, \gamma$  for UK.

		$\beta_{ki} = \bar{\beta}_k$	$\beta_{ki} = \bar{\beta}_k$	$\beta_{ki} = \bar{\beta}_k$	$\beta_{ki} = \bar{\beta}_k$
		$\beta_1 = \beta_2$	$\beta_1 = \beta_2$	$\beta_1 \neq \beta_2$	$\beta_1 \neq \beta_2$
Democratic Party	.const	-4.62 (0.13)	-4.71 (0.12)	-5.13 (0.09)	-1.23 (0.00)
	age	0.02 (0.32)	0.02 (0.35)	0.02 (0.28)	
	gender	-0.40 (0.52)	-0.40 (0.52)	-0.41 (0.51)	
	religion	-0.38 (0.04)	-0.36 (0.05)	-0.34 (0.06)	
	communist	1.22 (0.31)	1.21 (0.32)	1.24 (0.30)	
	education	0.43 (0.00)	0.44 (0.00)	0.44 (0.00)	
Social Democracy	.const	-0.37 (0.85)	-0.33 (0.87)	-0.90 (0.65)	-0.47 (0.04)
	age	0.02 (0.16)	0.02 (0.17)	0.02 (0.12)	
	gender	0.08 (0.87)	0.07 (0.88)	0.10 (0.83)	
	religion	-0.34 (0.03)	-0.34 (0.03)	-0.29 (0.06)	
	communist	-0.27 (0.70)	-0.30 (0.67)	-0.22 (0.75)	
	education	0.22 (0.02)	0.22 (0.02)	0.22 (0.02)	
Law and Justice	.const	2.76 (0.07)	2.86 (0.05)	2.50 (0.10)	1.70 (0.00)
	age	0.00 (0.97)	0.00 (0.98)	0.00 (0.94)	
	gender	-0.02 (0.94)	-0.04 (0.90)	-0.03 (0.92)	
	religion	-0.14 (0.28)	-0.14 (0.27)	-0.12 (0.34)	
	communist	-0.11 (0.84)	-0.12 (0.82)	-0.07 (0.89)	
	education	0.02 (0.78)	0.02 (0.83)	0.02 (0.73)	
Democratic Left	.const	1.65 (0.33)	1.91 (0.25)	1.15 (0.50)	0.31 (0.09)
	age	0.03 (0.04)	0.02 (0.04)	0.03 (0.03)	
	gender	0.08 (0.83)	0.08 (0.83)	0.10 (0.80)	
	religion	-0.43 (0.00)	-0.44 (0.00)	-0.39 (0.00)	
	communist	-0.36 (0.54)	-0.42 (0.47)	-0.31 (0.59)	
	education	0.10 (0.20)	0.09 (0.30)	0.10 (0.22)	
Civic Platform	.const	1.94 (0.23)	1.94 (0.22)	1.44 (0.37)	1.54 (0.00)
	age	-0.01 (0.52)	-0.01 (0.38)	-0.01 (0.62)	
	gender	-0.22 (0.50)	-0.27 (0.43)	-0.24 (0.47)	
	religion	-0.26 (0.04)	-0.25 (0.05)	-0.22 (0.08)	
	communist	0.66 (0.26)	0.65 (0.26)	0.69 (0.24)	
	education	0.12 (0.11)	0.13 (0.08)	0.12 (0.09)	
People's Party	.const	-0.36 (0.86)	-0.12 (0.95)	-0.62 (0.76)	-0.21 (0.28)
	age	0.00 (0.76)	0.00 (0.77)	0.00 (0.85)	
	gender	-0.40 (0.33)	-0.42 (0.32)	-0.41 (0.33)	
	religion	0.00 (1.00)	-0.01 (0.96)	0.03 (0.87)	
	communist	0.39 (0.59)	0.33 (0.66)	0.40 (0.58)	
	education	0.05 (0.60)	0.04 (0.65)	0.04 (0.65)	
Self Defense	.const	4.05 (0.02)	4.29 (0.01)	3.68 (0.04)	0.32 (0.08)
	age	-0.01 (0.32)	-0.01 (0.33)	-0.01 (0.41)	
	gender	-0.21 (0.57)	-0.19 (0.61)	-0.21 (0.58)	
	religion	-0.18 (0.19)	-0.18 (0.18)	-0.15 (0.29)	
	communist	-0.16 (0.80)	-0.24 (0.70)	-0.14 (0.82)	
	education	-0.27 (0.01)	-0.29 (0.00)	-0.28 (0.00)	
$\beta_1$	.const	0.49 (0.00)	0.11 (0.66)	0.41 (0.00)	0.45 (0.00)
	age		0.00 (0.27)		
	gender		-0.08 (0.17)		
	religion		0.01 (0.42)		
	communist		0.11 (0.28)		
	education		0.04 (0.00)		
$\beta_2$	.const	0.49 (0.00)		0.55 (0.00)	0.59 (0.00)
$\rho$	.const	2.35 (0.00)	2.08 (0.00)	2.49 (0.00)	2.41 (0.00)
$\gamma$	.const	1.45 (0.00)	1.67 (0.00)	1.57 (0.00)	1.59 (0.00)
Log likelihood		-1400.90	-1391.87	-1396.60	-1456.37
N		918	918	918	918

Table 7: Estimated coefficients with unrestricted  $\rho$ ,  $\gamma$  for Poland.

		$\beta_{ki} = \bar{\beta}_k$		$\beta_{ki} = \bar{\beta}_k$	
		$\beta_1 = \beta_2$	$\beta_1 = \beta_2$	$\beta_1 \neq \beta_2$	$\beta_1 \neq \beta_2$
United Russia	<code>_const</code>	2.88 (0.00)	2.87 (0.00)	2.87 (0.00)	1.95 (0.00)
	<code>age</code>	-0.04 (0.00)	-0.04 (0.00)	-0.04 (0.00)	
	<code>gender</code>	0.36 (0.12)	0.37 (0.12)	0.36 (0.12)	
	<code>education</code>	-0.63 (0.17)	-0.64 (0.15)	-0.63 (0.18)	
	<code>income</code>	-0.15 (0.84)	-0.14 (0.85)	-0.14 (0.85)	
	<code>efficacy</code>	0.54 (0.13)	0.56 (0.09)	0.54 (0.11)	
	<code>approve_putin</code>	0.98 (0.02)	0.98 (0.02)	0.98 (0.02)	
Communists	<code>_const</code>	0.53 (0.59)	0.45 (0.63)	0.52 (0.60)	0.15 (0.28)
	<code>age</code>	0.01 (0.31)	0.01 (0.29)	0.01 (0.31)	
	<code>gender</code>	-0.26 (0.38)	-0.27 (0.35)	-0.26 (0.38)	
	<code>education</code>	-0.07 (0.90)	0.01 (0.99)	-0.07 (0.91)	
	<code>income</code>	0.48 (0.60)	0.52 (0.57)	0.48 (0.60)	
	<code>efficacy</code>	-0.28 (0.52)	-0.34 (0.43)	-0.28 (0.52)	
	<code>approve_putin</code>	-1.21 (0.00)	-1.19 (0.00)	-1.21 (0.00)	
Fair Russia	<code>_const</code>	2.42 (0.03)	2.41 (0.03)	2.40 (0.03)	-0.43 (0.01)
	<code>age</code>	-0.06 (0.00)	-0.06 (0.00)	-0.06 (0.00)	
	<code>gender</code>	-1.52 (0.00)	-1.51 (0.00)	-1.52 (0.00)	
	<code>education</code>	-0.40 (0.58)	-0.39 (0.58)	-0.40 (0.59)	
	<code>income</code>	2.28 (0.04)	2.28 (0.04)	2.29 (0.04)	
	<code>efficacy</code>	-0.01 (0.99)	-0.01 (0.98)	-0.01 (0.99)	
	<code>approve_putin</code>	-0.93 (0.06)	-0.93 (0.06)	-0.92 (0.06)	
$\beta_1$	<code>_const</code>	0.46 (0.00)	0.51 (0.00)	0.46 (0.00)	0.43 (0.00)
	<code>age</code>		0.00 (0.35)		
	<code>gender</code>		0.02 (0.76)		
	<code>education</code>		-0.11 (0.25)		
	<code>income</code>		0.03 (0.86)		
	<code>efficacy</code>		0.08 (0.28)		
	<code>approve_putin</code>		0.01 (0.91)		
$\beta_2$	<code>_const</code>			0.46 (0.00)	0.51 (0.00)
$\rho$	<code>_const</code>	2.99 (0.01)	2.63 (0.00)	2.99 (0.01)	2.84 (0.00)
$\gamma$	<code>_const</code>	1.75 (0.00)	1.87 (0.00)	1.75 (0.00)	1.79 (0.00)
Log likelihood		-717.89	-715.36	-717.88	-795.10
N		1004	1004	1004	1004

Table 8: Estimated coefficients with unrestricted  $\rho$ ,  $\gamma$  for Russia.